Topic: Compound Interest and Depreciation **Name:** Solns

* Simple Interest (or Flat Rate Interest)

Simple interest is the interest which is paid to the investor and not added to the initial investment amount.

- The interest is not re-invested during the period of the investment.
- The interest paid to the investor is the same each year (or any time period) for the duration of the investment.

Any investment of $P at R% per annum (i.e. per year) simple interest for T years, the interest earned:

*

$$\text{Simple Interest (S.I.)} = \frac{PRT}{100}$$

where P = the principal

R = Rate (%) p.a. (p.a. means per annum or per year)

T = Number of years

- The **total amount** of the investment: Amount = Principal + Simple Interest

*

$$A = P + I$$

Note: For some textbooks: S.I. = P r t

where r needs to be converted to a decimal.

For example: 9% p.a. >>>>> r = 0.09

*** Flat rate loan:**

For a flat rate interest loan, the interest is calculated as simple interest for the full term of the loan and then this interest is spread equally over the number of repayments that are to be made to repay the loan.

$I = ?$

Ex 1. Calculate the **interest** earned on an investment of $900 at 4% p.a. simple interest invested for 3 years.

$$I = \frac{PRT}{100} = \frac{900 \times 4 \times 3}{100} = \$108$$

$R = ?$

Ex 2. A sum of $6,000 accumulates to $6,570 in 30 months. Calculate the **rate** of simple interest.

$$\begin{array}{r} 6570 \\ -6000 \\ \hline 570 \end{array}$$

$$I = \frac{PRT}{100}$$

$$570 = \frac{6000 \times R \times \frac{30}{12}}{100}$$

Solver: $R = 3.8\%$

$T = ?$

Ex 3. For **how long** must $10,000 be invested at 5.5% p.a. to earn $2,750 in simple interest?

$$I = \frac{PRT}{100}$$

$$2750 = \frac{10000 \times 5.5 \times T}{100}$$

Solver: $T = 5$ years

Ex 4.

A simple interest arrangement of 5·4% per annum sees an initial investment grow to $11121·60 in 6 years. What was the initial investment?

$P = ?$

$$I = \frac{PRT}{100}$$

$$11121.60 - P = \frac{P \times 5.4 \times 6}{100}$$

$$P = \$8400$$

Ex 5. Christine borrows $3,600 at a flat-rate of 6.6% p.a. The loan is to be repaid monthly over 18 months. Calculate

a) the **total interest** paid on this loan.

$$I = \frac{PRT}{100} = \frac{3600 \times 6.6 \times \frac{18}{12}}{100} = \$356.40$$

Note: 2 d.p. for money

b) the **monthly repayments** to be made.

Total $= 3600 + 356.40 = \$3956.40$

Monthly repayments $= \frac{3956.40}{18} = \$219.80$

✗ **Ex 6.**

Anna borrows \$6000 at a flat rate of 5.4% per year. The loan is to be repaid monthly over four years.
(a) Calculate the total interest that needs to be paid on this loan.

(b) Calculate the monthly repayments to be made.

✗ (c) Write a recurrence relation that will give the amount, a_n, Anna owes at the end of month n.

(d) How much does Anna owe after (i) 1 year? (ii) $2\frac{1}{2}$ years?

(e) At the start of which month does Anna owe less than \$1000 for the first time?

a) $I = \frac{PRT}{100} = \frac{6000 \times 5.4 \times 4}{100} = \$1296$

b) Total (Loan) = 6000 + 1296 = \$7296

Monthly repayments $= \frac{7296}{4 \times 12} = \frac{7296}{48} = \$152/\text{month}.$

c) $\underbrace{7296, 7144}_{-152}, \underbrace{7144, 6992}_{-152}, \ldots \Rightarrow$ A.P.

$\begin{cases} a_{n+1} = a_n - 152 \\ a_0 = 7296 \end{cases}$

or General rule

$a_n = a + (n-1)d$

$a_n = 7296 + (n-1)(-152)$

$a_n = 7448 - 152n$

d) i) 1 year ⇒ n = 12 months

Anna owed \$5472

ii) $2\frac{1}{2}$ years = 30 months

Anna owed \$2736

e)

n = 41 ⇒ A = \$1064

n = 42 ⇒ A = \$912

ie After 42 months

⇒ During the 43rd month

OR 1000 = 7448 − 152n

solver n = 42.42

n ≈ 43.

During the 43rd month.

Complete Ex2A

Compound Interest

When compound interest is calculated the interest earned during the first interest is added to the principal. It means that the first interest will be used to earn more interest for the following year. This process of adding the interest of one period to the principal at the end of each interest period is know as compounding the interest.

Earning interest on your interest as well as on your initial investment. An investment would be an exponential function since it represents a percentage increase situation. The amounts can also be thought of as **a geometric sequence.**

- **Total Amount**:

  Amount = $A = P(1+r)^t$, where P = Principal / the initial investment,

  r = interest rate (as a decimal), t = the time.

OR

- **Total Amount**:

  decimal

$$A = P(1+\frac{r}{n})^{nt}$$

where P = Principal / the initial investment,

r = interest rate (as a decimal), t = the time.

n = the number of times interest is added each year

- **Interest earned** = Amount – Principal = A – P = $P(1+\frac{r}{n})^{nt} - P$

Ex 7: Determine the interest paid on \$15,000 invested at 10% per annum **compound interest** for 3 years, with the interest compounded

P $r=0.1$

t

a) annually,
b) every six months,
c) quarterly,
d) monthly,
e) weekly,
f) daily.

$P = \$15000$
$r = 10\% = 0.1$
$t = 3$ years

a) Annually: $n=1$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 15000\left(1+\frac{0.1}{1}\right)^{(1\times3)}$

$A = \$19,965$

$\therefore I = A - P$

$I = 19965 - 15000 = \$4,965$

c) Quarterly: $n=4$

$A = 15000\left(1+\frac{0.1}{4}\right)^{(4\times3)} = \$20,173.33$

$I = 20173.33 - 15000 = \$5,173.33$

e) Weekly: $n=52$

$A = 15000\left(1+\frac{0.1}{52}\right)^{(52\times3)} = \$20,242.05$

$I = 20242.05 - 15000 = \$5,242.05$

b) Every six months: $n=2$

$A = 15000\left(1+\frac{0.1}{2}\right)^{(2\times3)} = \$20,101.43$

$I = 20101.43 - 15000 = \$5,101.43$

d) Monthly: $n=12$

$A = 15000\left(1+\frac{0.1}{12}\right)^{(12\times3)} = \$20,222.73$

$I = 20222.73 - 15000 = \$5,222.73$

f) Daily: $n=365$

$A = 15000\left(1+\frac{0.1}{365}\right)^{(365\times3)} = \$20,247.05$

$I = 20247.05 - 15000 = \$5,247.05$

$$\frac{3.8}{100} = 0.038$$

Ex 8. Find the value of an investment of $8,000 invested at 3.8% p.a. **compounded annually** for 5 years.

Method 1

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 8000\left(1+\frac{0.038}{1}\right)^{(1\times5)}$$

$$= \$9639.99$$

Method 2

$$A = P(1+r)^t$$

$$= 8000(1+0.038)^5$$

$$= \$9639.99$$

Ex 9. Calculate the interest earned after 3 years on an investment of $12,500 invested at 6.7% p.a. compounding **quarterly**.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 12500\left(1+\frac{0.067}{4}\right)^{(4\times3)}$$

$$= \$15257.39$$

* Interest earned:

$$I = A - P$$

$$= 15257.39 - 12500$$

$$= \$2757.39$$

Ex 10.

At an 8% annual compound interest rate, with compounding every six months, how many years would it take for an initial investment of $2500 to grow to $3700 ?

$n=2$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$3700 = 2500\left(1+\frac{0.08}{2}\right)^{(2t)}$$

solver: $t = 4.9979$

$t \approx 5$ years

Ex 11. An investment of $45,000 compounded quarterly, amounts to $53698.20 over a period of 5 years. How long, to the nearest month, will it take for the investment to amount of $60,000?

$n=4$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$53698.20 = 45000\left(1+\frac{r}{4}\right)^{(4\times5)}$$

solver: $r = 0.0355$

$r \approx 3.55\%$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$60000 = 45000\left(1+\frac{0.0355}{4}\right)^{4t}$$

Solver: $t \approx 8.1396$ years

$\approx$ 8 years 2 months

$(0.1396 \times 12 \approx 1.6752)$

Complete Ex 2B.

Compound interest invesments and loans using recurrence relations

Ex 12. Find the value of an investment of \$5,000 invested at 6% p.a. compounded annually for 5 years.

0.06

* Method 1

$A = P(1+r)^t$

$= 5000(1+0.06)^5$

$= \$6691.13$

* Method 2

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 5000\left(1+\frac{0.06}{1}\right)^{(1\times 5)}$

$= \$6691.13$

* Method 3

Begin \$5000

End of Year 1: $5000 \times 1.06 = 5300$

2: 5000×1.06^2

3: 5000×1.06^3

4: 5000×1.06^4

$\Rightarrow$ End of year 5: $\begin{cases} 5000 \times 1.06^5 \\ = \$6691.13 \end{cases}$

* Method 4: "Recursive"

$\begin{cases} T_{n+1} = T_n \times 1.06 \\ T_0 = 5000 \end{cases}$

$\Rightarrow$ find $T_5 = \$6691.13$

* Method 5

$T_{n+1} = T_n \times 1.06$

$T_1 = 5300$

$\Rightarrow$ Find $T_5 = \$6691.13$

* Method 6 "Explicit"

$T_n = 5300 \times 1.06^{n-1}$

$\Rightarrow$ Find $T_5 = \$6691.13$

$\nearrow 0.045$ $\nearrow n=12$

Ex 13. $25,000 is invested at 4.5% p.a. compounded monthly.

a) Calculate the value of this investment after 5 years.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 25000\left(1+\frac{0.045}{12}\right)^{(12\times 5)}$$

$$= \$\ 31294.90$$

OR Monthly Interest rate $= \frac{0.045}{12} = 0.00375$

monthly grow factor $= 1+0.00375 = 1.00375$

$$\begin{cases} T_{n+1} = 1.00375 T_n \\ T_0 = 25000 \end{cases}$$

$\Rightarrow$ find $T_{60} = \$31294.90$

b) How much interest is earned by this investment over the 5 years?

Interest $= I = A - P$

$= 31294.90 - 25000$

$= \$\ 6294.90$

c) Calculate the interest rate that would produce the same value for the given investment after 3 years.

$\longrightarrow 3\times 12 = 36$ months

$$25000 \times r^{36} = 31294.90$$

Solver:

$$r = 1.006257813$$

$\Rightarrow$ Interest rate as decimal $= 0.006257813 \times 12$

$= 0.075093756$

$\Rightarrow$ Interest rate as % $= 0.075093756 \times 100$

$= 7.5093756\ \%$

$\approx 7.51\%$

Ex 14. [Ex2C, Q1]

$2000 is invested at 8.4% per annum for 4 years compounded annually.

(a) Write a recurrence relation that describes the value of the investment at the end of n years.

(b) What is the value of this investment at the end of four years?

(c) How much interest has been earned after 4 years?

(d) How much interest has been earned after 3 years?

(e) What is the value of this investment after seven years?

(f) What interest rate will produce the same value in three years as that found in (e)?

a) $8.4\% = 0.084$

$T_{n+1} = 1.084\,T_n$

$T_0 = 2000$

b) Using Classpad,

$\Rightarrow$ Find $T_4 = \$2761.51$

c) Interest $= 2761.51 - 2000$

$= \$761.51$

d) After 3 years,

Total Amount (T_3) $= \$2547.52$

$\Rightarrow$ Interest $= 2547.52 - 2000$

$= \$547.52$

e)

Total Amount (T_7) $= \$3517.51$

f) $2000 \times r^3 = 3517.51$

Solver: $r = 1.20707739$

Interest rate as decimal $= 0.20707739$

Interest rate as $\% = 0.20707739 \times 100$

$\approx 20.7077\%$ (4dp)

$\approx 20.71\%$ (2dp)

Ex 15. [Ex2C, Q17]

A sum of $10 000 is deposited in an account. The terms of the account allow the value of the account to increase by 10% each year.

$a_{n+1} = 1.1 \times a_n$, $a_0 = 10000$

(a) Fill the table below to show the value of this investment over the first 5 years.

Time (t years)	0	1	2	3	4	5
Value ($V)	10 000	11 000	12100	13310	14 641	16105.10

(b) Write a recursive formula for V_t, the value of the term deposit after t years.

$V_{t+1} = 1.1\, V_t$

$V_0 = 10000$

(c) Show how to use your recursive formula to calculate the value of the investment after 6 years.

$V_6 = 1.1\, V_5$

$= 1.1 \times 16105.10$

$= \$17715.61$

$V_{16} = 45949.73$, $V_{17} = 50544.70$

(d) When will the value of the investment first amount to $50 000?

During the 17th year.

(e) How much interest will be earned by this investment after a period of 10 years?

$I = A - P = 25937.42 - 10000 = \$15937.42$

(f) How much interest is earned during the 10th year?

$I = A_{10} - A_9$

$= 25937.42 - 23579.48 = \$2357.94$

* (g) What simple interest rate will produce the same amount over a period of 10 years?

- Compound Interest

$t = 10: \; A = \$25937.42$

Interest: $I = 25937.42 - 10000 = \$15937.42$

* Simple Interest

$$I = \frac{PRT}{100}$$

$$15937.42 = \frac{10000 \times R \times 10}{100}$$

Solver: $R = 15.93742\%$

$R \approx 15.94\%$ (2 dp)

Complete Ex 2C

Using the "Financial Application" of a Graphic Calculator

Example. A sum of \$60,000 was invested at an interest rate of 6.5% p.a. compounding annually.

a) What is the value of this investment after 4 years?

b) How much interest was earned over the 4 years?

Solution

Classpad:

- Financial
- Compound Interest

For the situation under consideration:

N is the number of time periods, which in our case is 4 as we are compounding annually for 4 years.

I% is the annual interest rate as a percentage, which in our case is 6.5%.

PV is the present value or principal being invested and is entered as a negative quantity as we no longer have this money because we have given it to the bank. In our case we need to enter -60000.

PMT is the amount paid each period, for compound interest there is only one deposit which is the principal, hence we need to enter zero.

FV is the future value of the account, this the value of the investment and will be given as a positive quantity as this is the money that we get back from the bank.

P/Y is the number of instalment periods per year. In our case it is 1.

C/Y is the number of times interest is compounded per year. In our case it is 1 as interest is compounded annually.

For all compound interest calculations P/Y and C/Y take on the same value.

Compound Interest

N	4
I%	6.5
PV	-60000
PMT	0
FV	77187.98104
P/Y	1
C/Y	1

a) The value of this investment is given in the FV box:

= \$77, 187.98

b) Interest = Value of the investment – Amount invested

= \$77187.98 - \$60000

= \$17, 187.98

Ex 16. How much must be invested to amount to $21,000 in 4 years at 6.58% p.a. compounded monthly?

$\to n=12$

$16151.96 needs to be invested

Classpad

Compound Interest

N = 48 ← 4×12

I% = 6.58

⇒ PV = ? ⇒ −16151.96

PMT = 0

FV = 21000

P/Y = 12 ← "monthly"

C/Y = 12

Ex 17. [Ex 2D, Q4]

Heather has $12 000 to invest for 6 years and has a choice of the following investment options:

OPTION A: simple interest at 12.1% pa.
OPTION B: compound interest at 11.5% pa compounded annually.
OPTION C: compound interest at 10.6% pa compounded monthly.

(a) Which of these investments give the best return on her money.

Classpad

Option A

Simple Interest

Days = 365 ×6

I% = 12.1

PV = −12000

⇒ SI = ?? ⇒ 8712

SFV = ? ⇒ 20712

Interest = $8712

Option B

Compound Interest

N = 6

I% = 11.5

PV = −12000

PMT = 0

FV = ?? ⇒ 23058.47

P/Y = 1

C/Y = 1

Interest = 23058.47 − 12000
= $11058.47

Option C

Compound Interest

N = 72 ← 6×12

I% = 10.6

PV = −12000

PMT = 0

FV = ?? ⇒ 22603.71

P/Y = 12

C/Y = 12

Interest = 22603.71 − 12000
= $10603.71

⇒ { B is the Best option.

(b) What is the dollar value of this return?

From option B;

The dollar value of this return

= Interest

= $11,058.47

Complete Ex 2D

EFFECTIVE ANNUAL INTEREST RATE

Interest rates are usually given as a nominal (or stated) annual rate of interest. When compounding the interest occurs more than once per year, the actual rate of interest will be higher than the nominal rate of interest.

Consider an investment of $4000 invested at 9% per annum for 4 years compounded:

(a) annually
(b) quarterly
(c) monthly

Using the financial application the amount of each of these compoundings is given below.

(a) Compounding annually

Compound Interest	
N	4
I%	9
PV	- 4000
PMT	0
FV	5646.32644
P/Y	1
C/Y	1

(b) Compounding quarterly

Compound Interest	
N	16
I%	9
PV	- 4000
PMT	0
FV	5710.48583
P/Y	4
C/Y	4

(c) Compounding monthly

Compound Interest	
N	48
I%	9
PV	- 4000
PMT	0
FV	5725.621333
P/Y	12
C/Y	12

Examination of the answers for parts (a), (b) and (c) above clearly informs us that the nominal interest rate of 9% per annum applied over differing compounding periods does not give the same result.
It is quite obvious that the equivalent annual interest rate being applied in (b) and (c) above is more than the nominal 9% per annum. This equivalent interest rate is known as the **effective annual interest rate.**

NOTE:

- The **effective annual interest rate** = $(1+\frac{i}{n})^n - 1$

where i is the annual interest rate, written as a decimal

n is the number of compounding periods per year.

OR

- The **effective annual interest rate** = $i_{effective} = (1+\frac{i}{n})^n - 1$

Hence,

For the situation under consideration the effective annual interest rates can be calculated as follows:

(a) $i_{effective} = (1+\frac{i}{n})^n - 1 = (1+\frac{0.09}{1})^1 - 1 = 0.09$ that is 9% which is as expected because 9% is the annual rate.

(b) $i_{effective} = (1+\frac{i}{n})^n - 1 = (1+\frac{0.09}{4})^4 - 1 = 0.093083$ (6 d.p.) that is 9.3083% per annum.

(c) $i_{effective} = (1+\frac{i}{n})^n - 1 = (1+\frac{0.09}{12})^{12} - 1 = 0.093807$ (6 d.p.) that is 9.3807% per annum.

Ex 18.

Find the effective annual interest rate for a nominal interest of 8% per annum with compounding occurring quarterly.

$$i_{effective} = \left(1+\frac{i}{n}\right)^n - 1 = \left(1+\frac{0.08}{4}\right)^4 - 1 = 0.0824321 6$$

That is 8.2432% p.a.

Ex 19.

0.096

A sum of $10 000 is invested at a rate of 9.6% p.a. for 5 years compounded monthly.
(a) Calculate the interest earned if the interest is compounded monthly using a recurrence relation.
(b) Calculate the effective annual interest rate on this investment as a percentage rounded to 4 d.p.
(c) Calculate the interest earned on this investment using your calculated effective annual interest rate.
(d) Compare and comment on your answers to (a) and (c).

a) Interest as decimal $= \frac{0.096}{12} = 0.008$

Multiplier: $1 + 0.008 = 1.008$

$$\begin{cases} T_{n+1} = 1.008\,T_n \\ T_0 = 10000 \end{cases}$$

$\Rightarrow$ Find $T_{60} = \$16129.91$

Interest $= 16129.91 - 10000 = \$6129.91$

OR "Compound Interest"

N = 60

I% = 9.6

PV = -10000

PMT = 0

FV = ?? $\Rightarrow$ 16129.91

P/Y = 12

C/Y = 12

$\Rightarrow$ Total Amount = $16 129.91

Interest = 16129.91 − 10000

= $ 6129.91

b) $i_{effective} = \left(1+\frac{i}{n}\right)^n - 1$

$= \left(1+\frac{0.096}{12}\right)^{12} - 1$

$= 0.1003386937$

$\approx 10.0339\%$ (4 dp)

c) $1 + 0.100339$

$$\begin{cases} T_{n+1} = 1.100339\,T_n \\ T_0 = 10000 \end{cases}$$

$\Rightarrow$ Find $T_5 = \$16129.93$

Interest $= 16129.93 - 10000 = \$6129.93$

d) The answers to (a) and (c) should be the same.

However, the answers differ by 2 cents ($6129.93 − $6129.91 = $0.02) due to rounding of the effective annual interest rate to 4 d.p.

Complete Ex 2E

Inflation and Depreciation Inflation

Inflation is a term used by economists to describe an increase in the price that you pay for an item, or in other words it is a decrease in the purchasing power of money.

For example it the price of a litre of petrol today is \$1.50 and the inflation rate is 4% p.a. then the price of a litre of petrol one year from today will be 1.04 x \$1.50 = \$1.56, that is, an increase in price of 6 cents. Alternatively, \$1.50 will no longer buy a litre of petrol that is, the purchasing power of \$1.50 has been decreased.

Ex 20.

The present cost of an item is \$80. What will it cost in 6 years from now, if inflation is running at a constant rate of 3.8% p.a.?

0.038

Cost (after 1 year) = \$80 × 1.038

Cost (after 2 years) = 80 × $1.038^{\boxed{2}}$

Cost (after 3 years) = 80 × 1.038^3

⋮

Cost (after 6 years) = 80 × 1.038^6 = \$100.06

Note: $T_n = 80 \times 1.038^n$

or $\begin{cases} T_{n+1} = 1.038 T_n \\ T_0 = 80 \end{cases}$

Ex 21.

A home unit advertised for \$350 000 was purchased for \$280 000 five years ago. What constant annual rate of inflation would produce this price increase?

$280\,000 \times r^5 = 350\,000$

Solver: $r = 1.0456$ (growth factor)

$1.0456 - 1 = 0.0456 \Rightarrow 4.56\%$ increase

The inflation rate is 4.56%.

Ex 22. [Ex 2F, Q8]

What inflation rate needs to be applied to cause the value of a commodity to double in value

(a) in 5 years?

double original value

$2P = P \times r^5$

$2 = r^5$

solver: $r = 1.1487$

$\therefore$ The inflation rate is 14.87%

(b) in 10 years?

$2P = P \times r^{10}$

$2 = r^{10}$

Solver $r = 1.0718$

$\therefore$ The inflation rate is 7.18%

Complete Ex 2F

Depreciation

Depreciation is the reduction in the value of an item or asset usually due to age and/or wear and tear due to use. An estimate of the value of an item or asset is called the **book value** or the **written down value** and is calculated using the formula : Book value = Original cost of the item – Depreciation.

Book value = Original Cost – depreciation

The **residual value** of an item or an asset is the value of that item or asset **at the end of it useful life**, that is its salvage value, trade-in value or scrap value.

Depreciation can be calculated using three different methods:

- **straight-line (or prime cost or flat rate) depreciation method**,
- **unit cost method of depreciation,** and
- **reducing (or declining) balance depreciation method or diminishing balance depreciation.**

Flat Rate Depreciation Method

In **flat rate or straight-line** depreciation, the value of an asset is depreciated by a fixed amount each year. With straight-line depreciation the asset loses the same amount of value each year. In other words, this method spreads the cost of the item or asset evenly over its useful life.

The depreciation expense of an item is given by: $\text{Depreciation expense} = \dfrac{\text{Cost} - \text{Residual Value}}{\text{Useful life of the asset}}$

* The calculation of straight-line depreciation can be modelled by an arithmetic sequence.

Ex 23.

The purchase price of George's computer system was \$6800. George depreciates his computer system at a rate of 10% per annum, using the straight-line depreciation method.
(a) How much value will the computer system lose each year?
(b) Find the book value of the computer system after 8 years.
(c) When will the book value of the computer system be zero?

a) Yearly depreciation : 10% of \$6800

$= \frac{10}{100} \times 6800 = \$680$

b) Book value after $\boxed{1}$ year $= 6800 - \boxed{1} \times 680 = \$6120$

" " $\boxed{2}$ years $= 6800 - \boxed{2} \times 680 = \$5440$

⋮

∴ Book value after 8 years $= 6800 - 8 \times 680 = \$1360$

OR ⇒ A.P: $6120, 5440, \ldots$ (-680) $\begin{cases} a = T_1 = 6120 \\ d = -680 \end{cases}$

$T_n = a + (n-1)d = 6120 - (n-1)(-680) = 6800 - 680n$

$T_8 = 6800 - 680(8) = \$1360$

G.C. Explicit $a_n = 6800 - 680n \Rightarrow a_8 = 1360$

OR Recursive: $\begin{cases} T_{n+1} = T_n - 680 \\ T_0 = 6800 \end{cases} \Rightarrow$ Find $T_8 = \$1360$

c) $6800 - 680n = 0$
Solver $n = 10$ years. (or "written off")

Ex 24.

Cost — useful life — trade-in value

Machinery purchased for $300 000 has a lifetime of 16 years and has a residual value of $20 000. ← residual value
(a) What is the straight-line depreciation expense for this machinery?
(b) What is the value of this machinery after 10 years?

a) $$\text{Depreciation expense} = \frac{\text{Cost} - \text{Residual Value}}{\text{Useful life of the asset}}$$

$$= \frac{300000 - 20000}{16}$$

$$= \$17{,}500 \text{ per year.}$$

b) Initial value $= \$300{,}000$

After 1 year $= 300000 - 17500 = \$282{,}500$

" 2 years $= 300000 - 2 \times 17500 = \$265{,}000$

[or $282500 - 17500$]

" 3 years $= 300000 - 3 \times 17500 = \$247{,}500$

⋮

After 10 years $= 300000 - 10 \times 17500 = \$125{,}000$

OR $\begin{cases} T_{n+1} = T_n - 17500 \\ T_0 = 300000 \end{cases}$

$\Rightarrow$ find $T_{10} = \$125{,}000$

OR A.P.

$a = 300000 - 17500$

$a = \$282{,}500$

$T_n = a + (n-1)d$

$= 282500 + (n-1)(-17500)$

$= 300000 - 17500n$

$\Rightarrow T_{10} = 300000 - 17500(10)$

$= \$125{,}000$

Complete Ex 2G

Unit Cost Depreciation Method

In unit cost depreciation, the depreciation of the asset is based on the number of units produced by the asset during the year. This method of depreciation is sometimes referred to as **units of production method** or **units of activity method.**

Unit cost depreciation may be modelled by an arithmetic sequence in situations where the production level is the same each year over the useful life of the asset.

To calculate depreciation using the unit cost method:

1. Estimate the total number of units to be produced (for example the number of copies made using a photocopier, the number of kilometres driven by a car, the number of hours a machine was used etc.) over the useful life of the asset.

* 2. Calculate the depreciation cost per unit of production using the formula:

$$\boxed{\text{Depreciation cost per unit} = \frac{\text{Cost} - \text{Residual Value}}{\text{Total number of units produced}}}$$

* 3. Calculate the depreciation based on the number produced during the year using the formula:

$$\text{Depreciation expense} = \frac{\text{Cost} - \text{Residual Value}}{\text{Total number of units produced}} \times \text{Number of units produced}$$

Hence $\boxed{\text{Depreciation expense} = \text{Depreciation cost per unit} \times \text{Number of units produced}}$

Ex 25.

cost Residual value

A piece of machinery purchased for \$330 000 has a residual value of \$50 000 and is expected to produce 140 000 units over its useful life.

Total units

(a) If this machine produces 10 000 units in the first year, calculate the depreciation expense for the first year and the book value of the machinery after the first year.

Due to a steady demand for this product, the enterprise produces 10 000 units of this product every year.

(b) What will be the book value of the machinery after 10 years?

(c) When will book value reduce to the residual value?

a) Depreciation Cost per unit $= \dfrac{\text{Cost} - \text{Residual Value}}{\text{Total number of units produced}}$

$= \dfrac{330000 - 50000}{140000} = \$2$ per unit

Depreciation expense $=$ Depreciation cost per unit $\times$ No. of units produced

$= \$2 \times 10000 = \$20{,}000$

Book value after 1st year $= 330{,}000 - 20{,}000 = \$310{,}000$

b) Book value after 1 year $= 330000 - 1 \times 20000 = \$310{,}000$

" " " $\boxed{2}$ years $= 330000 - \boxed{2} \times 20000 = \$290{,}000$

10 years $= 330000 - 10 \times 20000 = \$130{,}000$

OR G.C. $\begin{cases} T_{n+1} = T_n - 20000 \\ T_0 = 330\,000 \end{cases} \Rightarrow T_{10} = 130000$

c) $50000 = 330000 - 20000 \times n$

solve $n = 14$ years

OR Sequence

$\Rightarrow$ Find $T_{14} = 50000$

**** Ex 26.**

At the start of the 2014 financial year Unitec purchased a van for $34 500. The van has been estimated to have a useful life of 300 000 km. The trade-in value of this van at the end of its useful life has been estimated to be $6000. The spreadsheet below shows the kilometres travelled over the first four years.

Year	Kilometres	Annual Depreciation	Accumulated Depreciation	Book Value
2014	85 000	8075	8075	26425
2015	68 000	6460	14535	19965
2016	75 000	7125	21660	12840
* 2017	84 000	(7980) 6840	(29640) 28500	(4860) 6000

Complete the spreadsheet using the unit cost depreciation method.

* Depreciation cost per km $= \dfrac{\text{Cost} - \text{Residual Value}}{\text{Total No. of units produced}}$

$= \dfrac{34500 - 6000}{300000} = \$0.095$ per km

[or 9.5 cents/km]

* Book value = Original Cost − Accumulated Depreciation

Using spreadsheet (G.C.)

	Col. A	Col. B	Col. C	Col. D
1	85 000	=0.095×A1 = 8075	= B1	= 34500 − C1
2	68 000		= C1+B2	
3	75 000			
4	84 000			

Completing the table using → Edit → Fill Range → ok

Notes Year 2017

- The Book value is $4860, which is less than $6000 (the trade-in value)

⇒ The correct book value = $6000

- The max. accumulated depreciation is $34500 − $6000 = $28500

⇒ The correct accumulated depreciation is $28500

- We have "over-depreciated" the asset by $6000 − $4860 = $1140

⇒ The correct annual depreciation = $7980 − $1140 = $6840

Complete Ex 2H

Reducing Balance Depreciation

With reducing balance depreciation the book value of an item is reduced by a fixed percentage of its book value at the beginning of each year. Hence the depreciation varies from year to year. Sometimes reducing balance depreciation is called **diminishing value** depreciation.

NOTE: If there is no indication in a problem that the flat rate depreciation method should be used we automatically assume the we should use the reducing balance depreciation method.

Ex 27.

The value of a new car is $26 000. Find the book value of this car after 4 years if it depreciates each year by 8% of its book value.

8% depreciation $\Rightarrow$ 92% value = 0.92

Book value after 1 year = 26000×0.92

" " " $\boxed{2}$ years = $26000 \times 0.92^{\boxed{2}}$

" " " $\boxed{3}$ years = $26000 \times 0.92^{\boxed{3}}$

" " " 4 years = 26000×0.92^4 = \$18,626.22

G.C. "Recursive"

$$\begin{cases} T_{n+1} = 0.92 T_n \\ T_0 = 26000 \end{cases}$$

$\Rightarrow T_4$ = \$18626.22

or

"Explicit"

$$T_n = 26000 \times 0.92^n$$

$\Rightarrow T_4$ = \$18626.22

Ex 28. [Ex 2I, Q23]

Frank's 5 year old car is currently valued at $12 310 and when it was 2 years old it was valued at $18 375.

(a) Calculate the constant rate of depreciation as a percentage correct to 1 decimal place.

Depreciation cost in 3 years = 18375 − 12310 = $ 6065

Cost of depreciation/year = $\frac{6065}{3}$ = 673.8\dot{3}$/year

$\frac{673.83}{18375} = 0.0367$

$1 - 0.0367 = 0.9633$

⇒ 96.3%

[0.9633257748]

(b) Calculate the price Frank paid for the car when it was new.

$12310 = 0.9633^5 \times \text{Price}$

Price for new = $\frac{12310}{0.9633257748^5} \approx \$14838.57$

(c) Calculate the value of Frank's car 2 years from now.

$12310 \times 0.9633257748^2 = \$11423.64$

(d) Write a recursive formula which will give the value of Frank's car V after n years.

$\begin{cases} T_{n+1} = T_n \times 0.9633 \\ T_0 = 14838.57 \end{cases}$

(e) Using your formula or otherwise find after how many years will the value of Frank's car be less than $5 000 for the first time.

G.C.

$T_{29} = 5021.28$

$T_{30} = 4837.12$

Frank's car will be less than $5000 [after 30 years] during the 30th year.

Complete Ex 2I